

# DEPARTMENT OF MATHEMATICS

## S6 MATHEMATICS TEST 3 2026

**TIME: 2 HOURS 20 MINUTES**

**Answer all the four (4) items.**

### ITEM 1

A physics student at a secondary school in Kampala is investigating oscillatory motion in a laboratory experiment. He found out that the vertical displacement of the particle from its equilibrium position is modelled by  $y = 5 \sin 2t$ , where  $y$  is in centimetres and  $t$  is time in seconds.

The energy stored in the system at any instant is equal to the square of the displacement.

#### TASK

- Write down an expression for the energy function,  $E$ .
- Determine the total energy generated over one complete cycle of motion, that is from  $t = 0$  to  $t = 2\pi$ .
- Also, find  $\int_0^{\frac{\pi}{2}} 5 \sin 2t \cos 2t dt$ .

### ITEM 2

A civil engineer is tasked with calculating the total energy dissipated by a non-linear damping system over a specific time interval  $[1, 4]$ . The energy dissipation rate is modelled by the function  $f(x) = x^2 \ln x$ , where  $x$  represents time in seconds. The engineer wishes to use numerical integration to estimate the total energy dissipated.

#### TASK

- Use the trapezium rule with seven ordinates to estimate the total energy,  $\int_1^4 x^2 \ln x dx$ , correct to three decimal places.
- Compute the exact value of  $\int_1^4 x^2 \ln x dx$ , correct to three decimal places.
- Help the engineer to calculate the percentage error made in the estimation in (a).

### ITEM 3

A certain company manufactures electronic components. The engineer in charge of production modelled the age limit of the commodity by a continuous random variable  $Y$  (in years), defined by the probability density function

$$f(y) = \begin{cases} ky; & 0 < y < 2 \\ 2 - 2ky; & 2 < y < 3. \\ 0; & \textit{elsewhere} \end{cases}$$

It was also found out that  $Y$  is normally distributed with the same mean and standard deviation of the function that was modelled.

### TASK

- (a) Help the engineer to determine the mean and standard deviation of the distribution.
- (b) Find how many commodities, out of 200 selected commodities, would be found to have life limit between 1 and 2.5 years.

### ITEM 4

- (a) In a high – speed racing test, a car is accelerating uniformly along a straight track, with marked points A, B and C. The car takes 10 seconds to travel between points A and B which are 540m apart, and it then takes 15 seconds, to travel between points B and C, which are 585m apart. Assuming the car continues with the same pattern of uniform acceleration, determine how far beyond point C, the car will come to complete stop.
- (b) A truck of mass 1000kg rests on an inclined plane in a garage, with the plane tilted at  $40^\circ$  to the horizontal. The truck is attached by a light, inextensible cable that runs over a frictionless pulley mounted at the top of the incline. On the other side of the pulley, a particle of mass 400kg hangs freely, pulling the cable to draw the truck up the incline. When the system is released from rest, determine the force exerted by the cable on the pulley.

END