

DEPARTMENT OF MATHEMATICS

S6 MATHEMATICS TEST 4 2026

TIME: 2 HOURS 20 MINUTES

Answer all the four (4) items.

ITEM 1

A prestigious hotel in Kampala is upgrading its security system for a cashier's safe, which uses a combination lock and an access – code generator based on mathematical models. The management wants to optimize the safe's opening procedure and the engineer models the safe's PIN as a three – digit number given by $C = 100a + 20b + c$ where a , b and c are digits determined by solving the polynomial remainder problem involving the expression $P(x) = x^3 - 2x^2 + ax + b$, and the remainder when $P(x)$ is divided by $x - 2$ exceeds the remainder when $P(x)$ is divided by $x - 1$ by 1 and the same expression gives a remainder of 24 when divided by $x - 3$.

The value of c is the coefficient of x^3 in the expansion of $\sqrt{1 - 4x}$.

TASK

- Use the remainder theorem to find the constants a and b .
- Use the binomial theorem to expand $\sqrt{1 - 4x}$, as far as the term in x^3 .
- Identify the safe's PIN.

ITEM 2

A water supply project in a growing town council in Uganda requires to design an open – top cuboidal tank whose volume must be maximized while the external surface area is limited by the available sheet metal. The engineer will use calculus of small changes to assess how the volume reacts to tiny dimensional adjustments and will also model the construction cost by an exponential function.

The side length of the tank is x and the height is h and the total surface area of the tank

is 200 m^2 , and the cost function in shillings is $C(x) = 150000e^{0.01x}$.

At the opening ceremony for the new project, the guest of honour had to be served a champagne drink in a champagne glass which is in the shape of an inverted cone of depth 12 cm and radius 4 cm . Champagne is poured into it at the rate of $4\pi \text{ cm}^3 \text{ s}^{-1}$.

TASK

- (a) Help the engineer to determine the dimensions of the tank required for maximum volume.
- (b) Find the rate of change of the cost when $x = 2$.
- (c) Find the rate at which the depth of the champagne in the glass is increasing 3 seconds after pouring has commenced.

ITEM 3

A manufacturing company produces balls of strings for industrial use. Due to variations in the production process, the length of strings in a randomly chosen ball of strings can be modelled by a normal distribution with unknown mean μ and standard deviation σ . It was ascertained that 95% of the balls of strings have lengths exceeding 495 m and 99% have lengths exceeding 490 m.

TASK

- (a) Help the quality assurance manager to find:
 - (i) the mean and standard deviation of the lengths of the strings.
 - (ii) the probability that a ball picked at random has length between 502m and 513m.
- (b) Determine the probability, that in 10 selected balls, exactly 3 will have length below 485m.

ITEM 4

At a mechanical workshop in Bwaise, a technician is tasked with analyzing two separate force systems to ensure the stability of newly designed structural components. A robotic arm applies two concurrent forces on a joint. Force P has a magnitude of magnitude 26 N and acts in the direction of the unit vector $\frac{1}{13}(5\mathbf{i} + 12\mathbf{j})$

. Another force Q of magnitude 25 N acts parallel to the vector $4\mathbf{i} - 3\mathbf{j}$. The technician needs to determine the magnitude and direction of the resultant of the forces P and Q, in order to select an appropriate reinforcement material.

The technician also studies a planar trapezoidal frame ABCD in which $AB = 7\text{m}$, $AD = DC = 3\text{m}$.

Five forces of magnitudes 30 N, 50 N, 30 N, 20 N and $20\sqrt{2}\text{ N}$ act on the frame along AB, BC, CD, DA and AC respectively, their directions being represented by the order of the letters. Taking AB and AD as the horizontal axis and vertical axis respectively, the technician wishes to assess whether the frame will withstand the loading conditions without deforming.

TASK

- (a) Determine the magnitude and direction of the forces P and Q.
- (b) For the trapezoidal frame ABCD, find the magnitude of the resultant force and its inclination to AD..

END