DEPARTMENT OF MATHEMATICS

S6 MATHEMATICS TEST 5 2026

TIME: 2 HOURS 20 MINUTES

Answer all the four (4) items.

ITEM 1

A High Tech firm was hired to investigate a criminal case involving an accountant previously working at an NGO. The managers wanted to access the safe that was used by the accountant. The accountant had modelled the safe's PIN as a three – digit number given by $T = \frac{I}{P}$.

The accountant had determined I as the term independent of x in the expansion of $\left(3x^2 - \frac{4}{x}\right)^9$ and $P = (a \times b)^4$ where a and b are integers for which $x^3 + ax^2 + bx - 12$ is divisible by $x^2 - 4$.

TASK

(a) Use the remainder theorem to find the constants a , b and I.

(b) Help the firm specialist to obtain the safe's PIN.

ITEM 2

A farmer in a rural district wants to design an experimental agricultural plot that maximizes crop yield by optimizing the usable area. He has 1 km length of fencing material and plans to construct a rectangular enclosure adjacent to an existing permanent wall. He hires an engineer and he shares with him his plan.

The construction engineer is inspecting a buried sewer line that runs through a slopping underground tunnel in the area. The angle of inclination θ of the sewer pipe affects the flow rate, and the engineer models the relationship between the pipe's rise y and the reference angle x by the inverse trigonometric equation $y = \tan\left(2\tan^{-1}\frac{x}{4}\right)$.

(a) Help the engineer to determine the dimensions of the plot necessary for maximum area and find this maximum area.

(b) Help the engineer to deduce that $\frac{dy}{dx} = \frac{8\left(1 + y^2\right)}{16 + x^2}$, and calculate the flow rate when $x = \frac{\pi}{36}$ and $y = 0.2\,m$.

ITEM 3

A textile factory manufactures reels of yarn for industrial use. The length of yarn in a randomly selected reel is modelled by a normal distribution.

Production records show that 90% of the reels have yarn lengths exceeding 810 metres and 98% have yarn lengths exceeding 800 metres.

In a separate inspection process, the factory checks batches of 150 reels to see if the yarn length exceeds a specification of 815 metres. Past data indicate that 75% of the reels meet this specification. The production engineer wishes to use the normal approximation to the binomial distribution for analysis..

TASK

(a) Help the production manager to find:

(i) the mean and standard deviation of the lengths of the yarn lengths,

(ii) the probability that a randomly picked reel has length between 820m and 830m.

(b) Determine the probability that at least 110 reels in a batch of 150 reels meet the minimum length specification of 815m.

ITEM 4

In a mechanical workshop, the technician places a rigid body of mass 4 kg on a frictionless horizontal workbench.

Three forces $\mathbf{F}_1 = 2a\,\mathbf{i} + (5 - c)\,\mathbf{j}$, $\mathbf{F}_2 = 4\mathbf{i} + 7\,\mathbf{j}$ and $\mathbf{F}_3 = (3 + a)\mathbf{i} + 3\,\mathbf{j}$ are subjected onto the body. It was found out that the acceleration of the body is $-2\mathbf{i} + \mathbf{j}$.

The technician also wanted to ascertain the resultant of the forces that act on a uniform hexagonal plate ABCDEF. Forces of 20N, 30N, 40N, 25N and 20N where subjected to the plate in the directions AC, AD, CD, EF and AF respectively.

TASK

(a) Help the technician to find the values of *a* and *c*.

(b) Find the magnitude of the force $\mathbf{F}_1$.

(c) Determine the magnitude and direction of the resultant force on the hexagonal plate.

END