

DEPARTMENT OF MATHEMATICS

S6 MATHEMATICS TEST 6 2026

TIME: 2 HOURS 20 MINUTES

Answer all the four (4) items.

ITEM 1

In a robotic navigation module, an angular control system uses a composite trigonometric function to determine the input angle or the orientation, x , of a sensor arm. The system must satisfy the equation

$$\sin^2 2x - 3\sin 2x \cos 2x - 14\cos^2 2x = 0.$$

The engineers need to find the values of $0 \leq x \leq \pi$.

Additionally, the navigation algorithm involves a sensor fusion step that

requires proving the inverse trigonometric function $\tan^{-1} \frac{1}{3} + \tan^{-1} \frac{1}{4} + \tan^{-1} \frac{2}{9} = \frac{\pi}{4}$

and proving the equation $\frac{a^2 - b^2}{c^2} = \frac{\sin(A - B)}{\sin(A + B)}$, where A, B and C are angles of

a triangle ABC.

TASK

(a) Help the engineer to find the values of x .

(b) Prove that $\tan^{-1} \frac{1}{3} + \tan^{-1} \frac{1}{4} + \tan^{-1} \frac{2}{9} = \frac{\pi}{4}$.

(c) Show that for the triangle ABC, $\frac{a^2 - b^2}{c^2} = \frac{\sin(A - B)}{\sin(A + B)}$.

ITEM 2

Students in a civil engineering faculty found out that the deflection x of a loaded beam satisfies the equation $\ln x + 5x - 7 = 0$. They needed to show that the root of the equation was between 1 and 2 and then use the technique of linear

interpolation and Newton Raphson's method to determine the root correct to three decimal places.

TASK

- (a) Help the students to show that the root lies between 1 and 2, and hence use linear interpolation once to find an estimate to the root correct to 2 decimal place.
- (b) Guide the students to derive the simplest the simplest formula based on Newton Raphson's method for solving the equation $\ln x + 5x - 7 = 0$ and hence find the root correct to three decimal places.

ITEM 3

A coffee factory in Kiboga tracks the weights (in kg) of daily coffee production. A sample of 30 days gave a mean production of 850 kg. A 90% confidence interval for the mean daily production was found to be [820, 880]. The production manager wishes to find the standard deviation and the 94.5% confidence limits for the mean daily production of coffee.

The manager found out that 60% of their coffee beans meet export quality and he took a sample of 200 beans.

TASK

- (a) Help the production manager to determine
 - (i) the standard deviation of the coffee production
 - (ii) the 94.5% confidence limits for the mean daily coffee production.
- (b) Calculate the probability that out of 200 coffee beans considered, at least 125 beans meet export quality.

ITEM 4

During a regional car rally competition, notable drivers are driving in cars A and B.

At a certain moment, car A is 1 km from the finishing point and travels at 35 ms^{-1} with a uniform acceleration of 0.4 ms^{-2} while car B is 200 m behind car A and is moving at a velocity of 44 ms^{-1} with a uniform acceleration of 0.5 ms^{-2} .

It is required to determine the time taken before car B catches up with car A from the instant they are at the same starting point.

Another driver in car P had his car of mass 800 kg breakdown and it was repaired from a sloping friction free table inclined at angle 30° . The car P was connected by a light inextensible string passing over a pulley fixed at the top of table, to a load of 1200kg which hangs freely. The technician needed to know the resultant of the force at the pulley.

TASK

- (a) Determine the distance car B moves before overtaking car A.
- (b) How long does car B take waiting for car A at the finishing point?
- (c) Help the technician to determine the acceleration of the car P along the table and the force exerted by the string on the pulley.

END